Beam-Beam Simulations for Lepton Machines

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- Motivation for simulations from observed effects
- Methods—

  algorithms,

  simulation codes,

  and some results.
- Conclusions

Motivation for simulations from observed effects

• Luminosity in circular e^+e^- colliders is limited by the beam-beam effect.

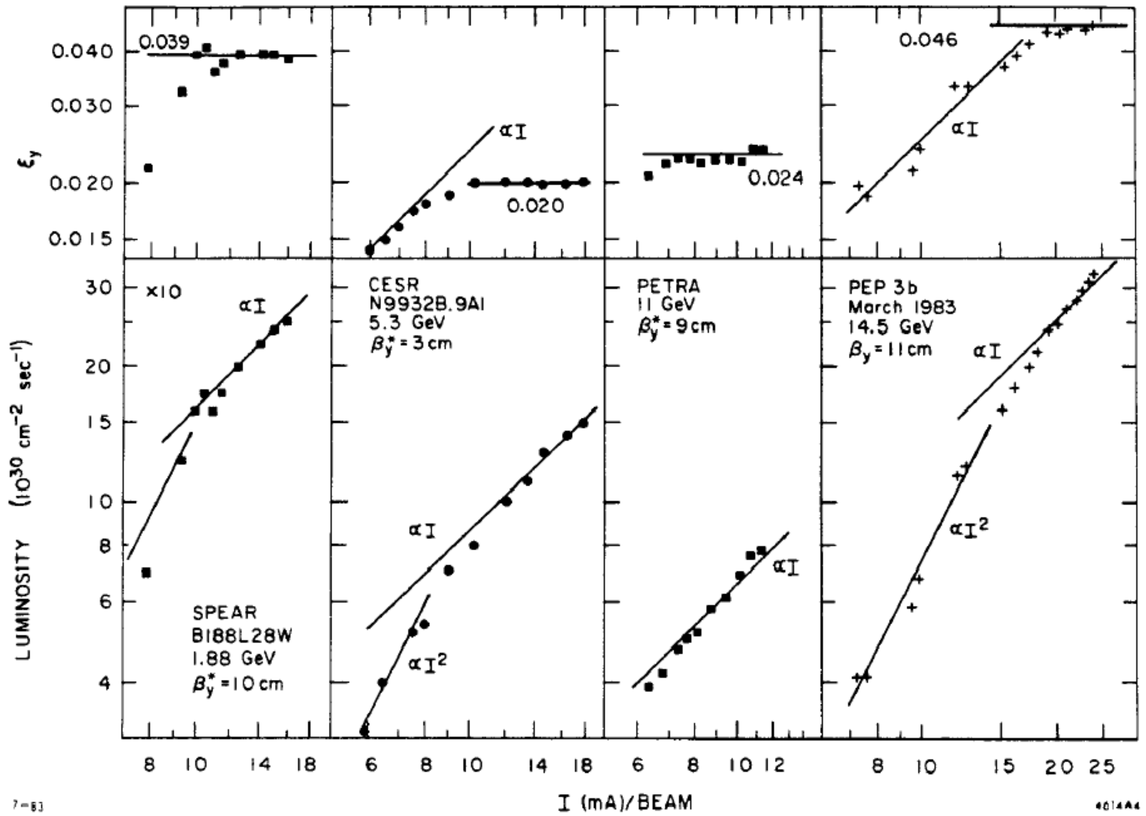
• There are two observed types of beam-beam limits when the bunch current is increased:

> First beam-beam limit: beam-beam parameter ξ_V or ξ_H reaches a limit as the core size starts to grow.
>
> Second beam-beam limit: a halo forms and beam loss increases.

• Circular e^+e^- colliders are also subject to the long-range beam-beam interaction.

Motivation for simulations from observed effects

First beam-beam limit (J. Seeman, 1983)

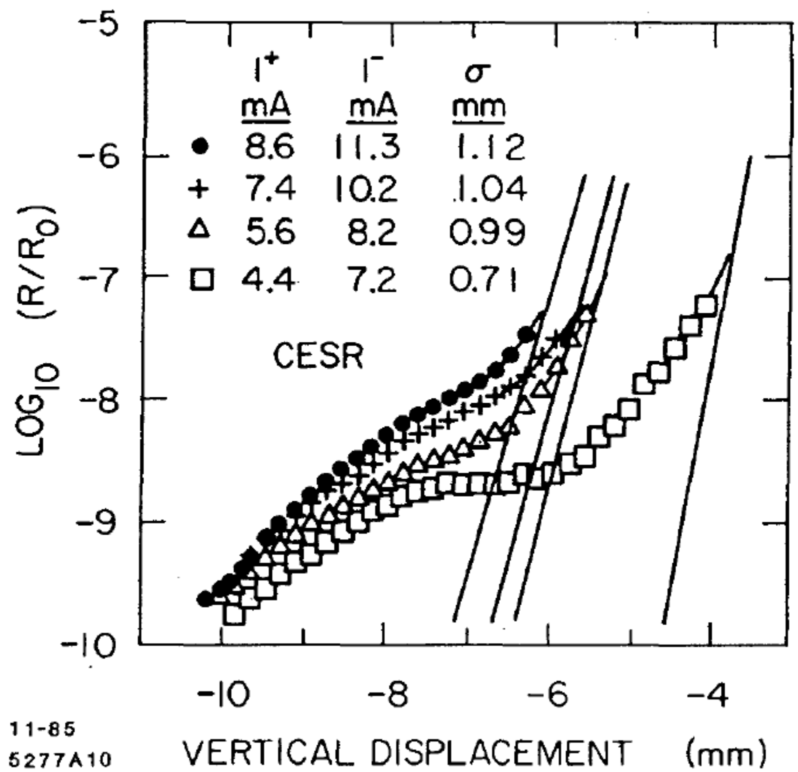


Luminosity and vertical tune shift parameter *vs*. beam current for SPEAR, CESR, PETRA & PEP.

Motivation for simulations from observed effects

Second beam-beam limit

(G. Decker, R. Talman, 1983)



Rate of bremsstrahlung photons produced by a thin Be wire in the vertical tail of the CESR beam. R_0 is the rate at the center of the beam, and the solid lines are extrapolations of the Gaussian core.

Motivation for simulations from observed effects

Two possible classes of causes of the beam-beam limits:

• Incoherent particle motion.

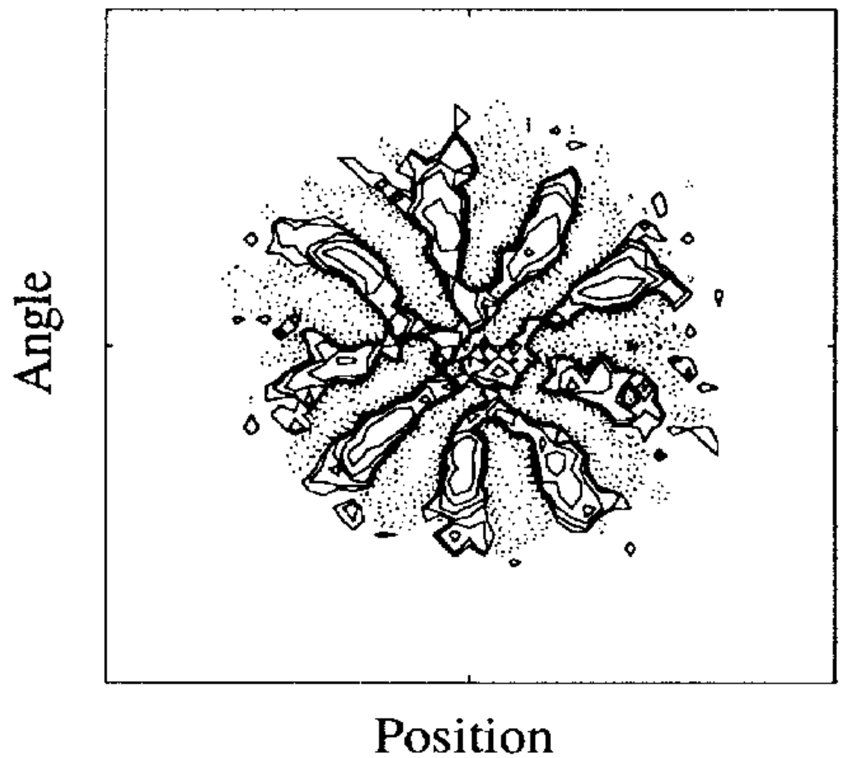
• Coherent particle motion. Evidence for a coherent beam-beam limit comes from:

Charge neutralization experiment at DCI. No increase in limiting ξ was observed.

Simulation results.

Motivation for simulations from observed effects

Four-beam charge compensation simulation (B. Podobedov, R. Siemann, 1995)



Horizontal contours of the difference of the phase-space density for two co-propagating beams showing an 18th order (!) resonance.

Motivation for simulations from observed effects

Is the first beam-beam limit caused by coherent motion? Or incoherent?
What about the second beam-beam limit?

	coherent	incoherent
first beam-beam limit	?	?
second beam-beam limit	?	?

Methods

Two classes of simulation methods (with many possible variations):

- Weak-strong simulations—

  One beam ("strong") is modeled as a fixed charged distribution. It serves as the source of the electromagnetic field that perturbs the particle distribution in the other ("weak") beam.

  Can simulate incoherent effects only.

- Strong-strong simulations—

  Both beams serve as the source of the field that perturbs the particle distribution in the other beam. This type of simulation is self-consistent.

  Can simulate coherent (and incoherent) effects.

Methods: weak-strong simulations

- Advantages:

  Electromagnetic field calculation need only be done once (because source distribution does not change from turn to turn). These codes are fast.

  Because the code is fast, many macroparticles may be tracked for many turns. Useful for halo/lifetime calculations (second beam-beam limit).

- Disadvantages:

  Sensitive only to incoherent effects.

  Not self-consistent.

Computation speed is a concern. Even though weak-strong codes are relatively fast, it is difficult to calculate lifetimes ~ hours without clever tricks!

Methods: weak-strong simulations

*(Note: don't even **think** about tracking the actual* ~ 10^{10} *particles in a beam!)*

• In a typical e^+e^- collider a beam-beam lifetime ~ 1 hour corresponds to a particle lifetime ~ 10^9 turns.

• Tracking for several$\times 10^9$ particle-turns can be too slow— for now.

• Tricks involving variable particle number are used (examples to follow).

Methods: weak-strong simulations—tricks of the trade

Crossing angle collisions (BBC, K. Hirata):

- 6-D symplectic code.
- Crossing angle implemented by a transverse Lorentz boost to a frame in which the beams collide head-on. Inverse boost to return to lab frame.
- Has been widely used.

Methods: weak-strong simulations— tricks of the trade

Halo distributions (J. Irwin, 1989) using “leap-frog” method:

• Establish a boundary in *H*-*V* amplitude space in which, *e.g.*, 90% of the initial distribution of *n* particles lies.

• Track the *n* particles for one damping time.

> ❶ Save the coordinates of a randomly selected particle outside the boundary every few turns.
>
> ❷ Save the coordinates of outward passages of particles across the boundary.

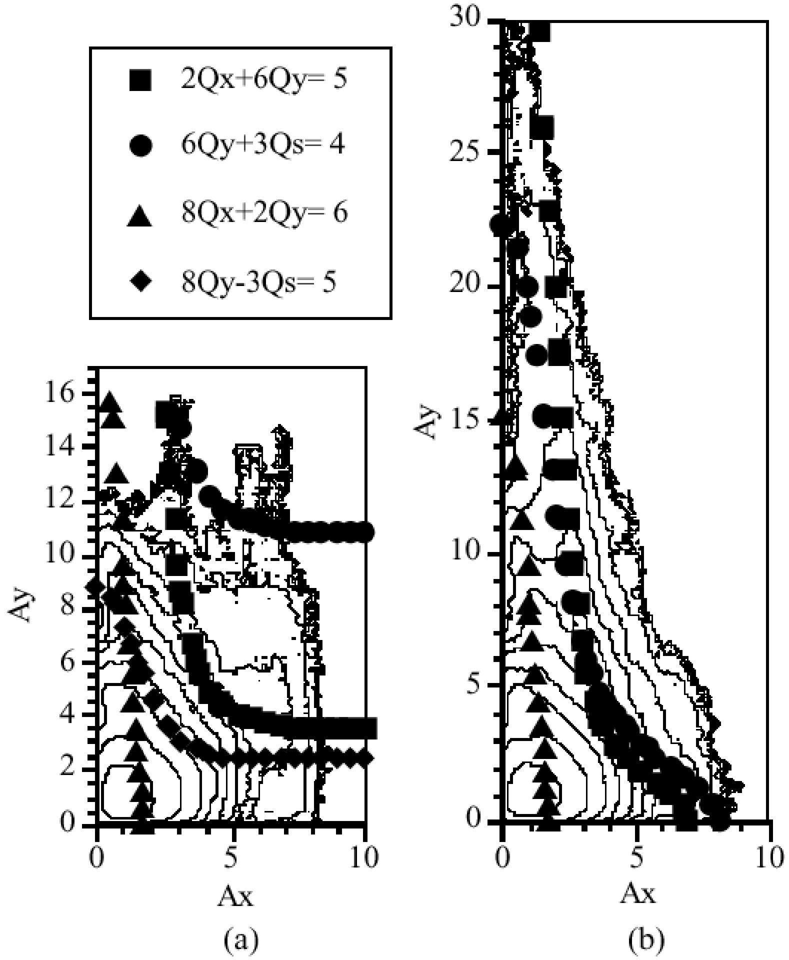
• Use ❶ to generate *n* new particles in the region outside the boundary.

• Track these *n* particles. When a particle passes inward through the boundary, replace it with a particle with coordinate from ❷.

This method is extended using multiple boundaries.

Methods: weak-strong simulations— tricks of the trade

Halo distributions using “leap-frog” method (T. Chen, J. Irwin, R. Siemann):



The beam tail distributions of the PEP-II HER (a) with linear lattice, and (b) with nonlinear tune shift with amplitudes $dQ_x/dA_x^2 = -85.6$, $dQ_x/dA_y^2 = dQ_y/dA_x^2 = -3931$, $dQ_y/dA_y^2 = -2.1$.

Methods: weak-strong simulations— tricks of the trade

Beam-beam interaction with rare scattering processes (*e.g.*, beam-gas scattering) (E.-S. Kim, K. Hirata, 1998):

- Distribution has "macroparticles" i =1, 2, …, n.
- Each macroparticle i has N_i particles.
- Particles undergo scattering randomly.
- When a particle in macroparticle i scatters

  a new (n+1)-th macroparticle is created with N_{n+1} = 1 particle,

  leaving N_i –1 particles in macroparticle i.

Methods: strong-strong simulations

- Advantages:

  Sensitive to both coherent and incoherent effects.

  Self-consistent. Useful for first beam-beam limit.

- Disadvantages:

  Electromagnetic field calculation must be done repeatedly. These codes are much slower than weak-strong codes.

  Because the code is relatively slow, fewer macroparticle-turns may be tracked. Limited usefulness for halo/lifetime calculations.

Computation speed is a serious hurdle. Strong-strong simulations require clever tricks to evolve the beam for several radiation damping times.

Methods: strong-strong simulations— types

- Particle-particle method:

  Pairwise interaction between particles. Computation time scales as n^2—too slow to be practical.

- Dynamic Gaussian models:

  Macroparticle distribution is fit by a Gaussian charge distribution, which serves as the source for the field that perturbs the other beam. The 1st moments (rigid-Gaussian) or 1st and 2nd moments (soft-Gaussian) are free to evolve.

- Particle-in-cell (PIC):

  Track "macroparticles" moving under the influence of the beam-beam force. The electromagnetic field is calculated on a discrete grid after assigning the macroparticle charge to grid points. The field at the location of the counter-rotating macroparticles is determined by interpolation from the grid.

Methods: strong-strong simulations— types

- Quasi-strong-strong:

  Role of weak and strong beam are exchanged periodically. Self-consistent on long time scales.

- Fast multipole method:

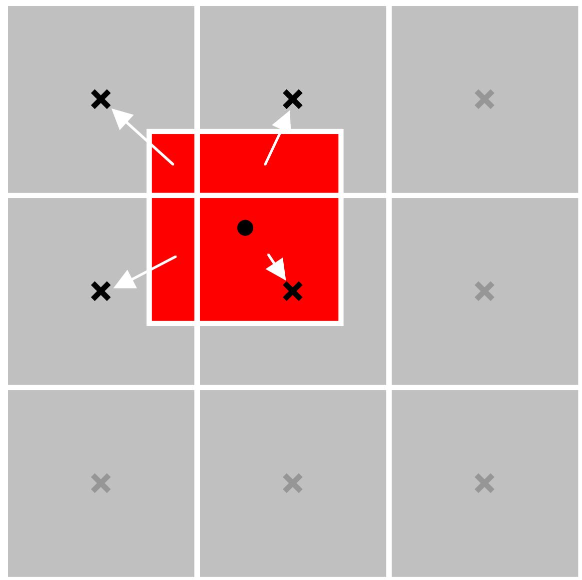
  Potential due to distant source particles is calculated as a multipole expansion.

- Numerical Vlasov equation solver:

  Evolves phase space density.

Methods: strong-strong simulations— particle-in-cell

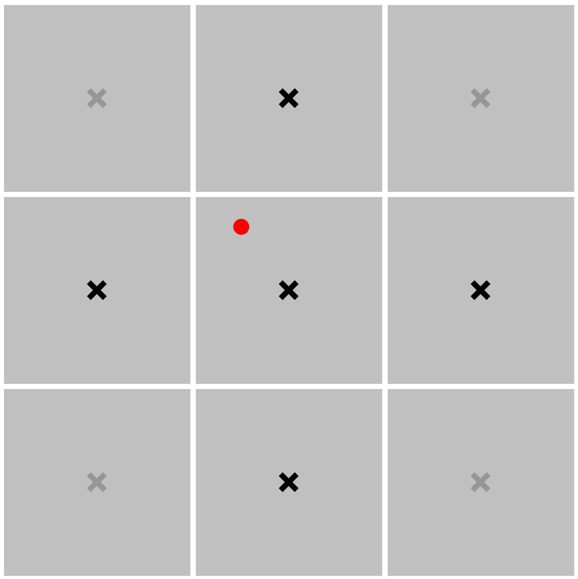
Assigning macroparticle charge to the grid: 4-point cloud-in-cell



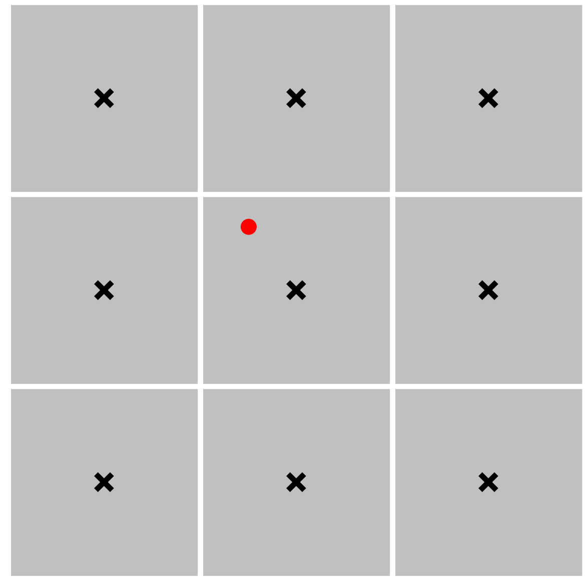
× = grid point

Methods: strong-strong simulations— particle-in-cell

5-point charge assignment



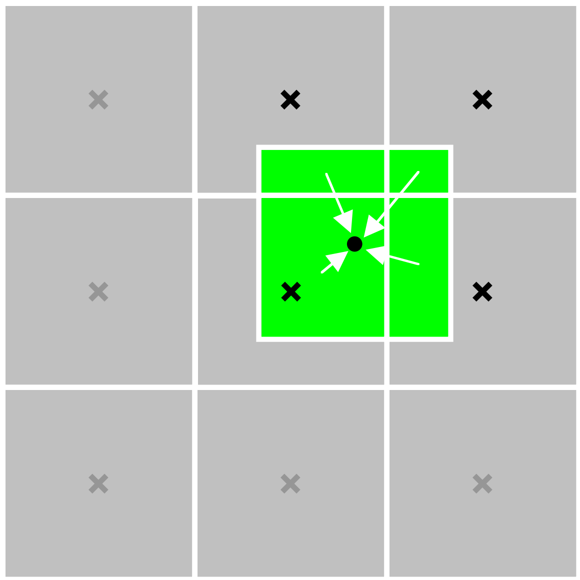
9-point charge assignment



Methods: strong-strong simulations— particle-in-cell

• Field at position of perturbed particle must determined by interpolation between grid points.

• Interpolation must use the same weighting as charge assignment to conserve the transverse momentum of the beams.



× = grid point

Methods: strong-strong simulations— PIC tricks of the trade

Fast Poisson solver (S. Krishnagopal, 1996, and earlier work with R. Siemann)

• Solution of Poisson equation on a Cartesian grid by Fast Fourier Transform (FFT) and cyclic reduction (FACR) method.

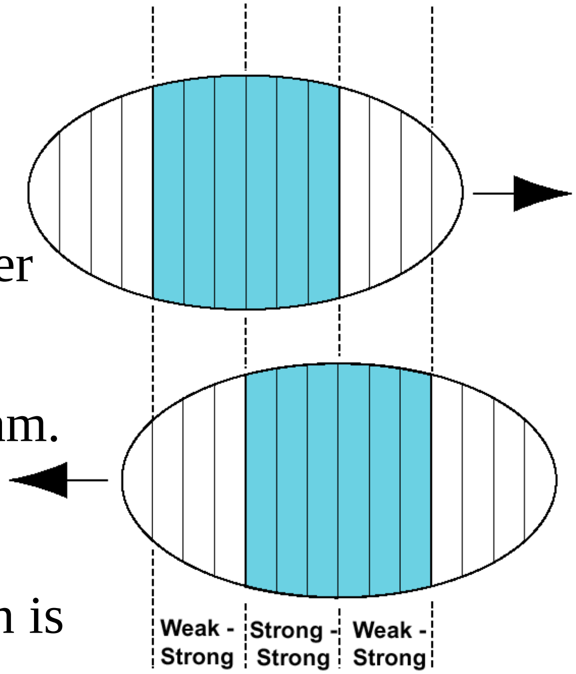
• Coherent phenomena seen in simulation results:

Flip-flop instability (stationary equilibrium with unequal beam sizes).

Period-*n* anticorrelated beam size oscillations.

Methods: strong-strong simulations— PIC tricks of the trade

Adaptive Green's function method with longitudinal dynamics (ODYSSEUS, E. Anderson, J.R., 1999)



Goal was to develop a fast strong-strong code with longitudinal dynamics. Particles are sorted into longitudinal "slices".

• Grid periodically adapts size and aspect ratio to cover beams (except transverse tails).

• Transverse tails use a soft-Gaussian fit to source beam.

• Longitudinal tails are weak-strong.

• Convolution of charge density with Green's function is done in Fourier coefficient space. A sharpening function is included in the convolution to counter the "low-pass" effect of charge assignment.

Methods: strong-strong simulations— PIC tricks of the trade

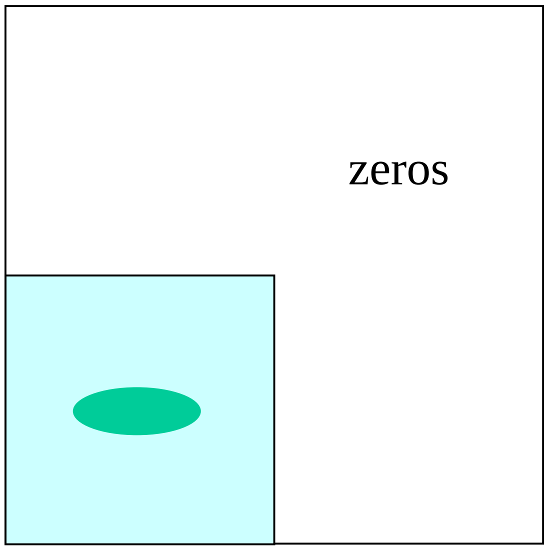
Convolution of Green's function and charge density

• Brute-force convolution time scales as $N_g^{\,2}$ (N_g = number of grid points).

• Cylic convolution can be done as a multiplication of Fourier series coefficients.

• FFT and inverse FFT dominate computation time but scale as $N_g \log N_g$— better!

• Cyclic convolution requires a grid that is 4× larger than the area occupied by charge:

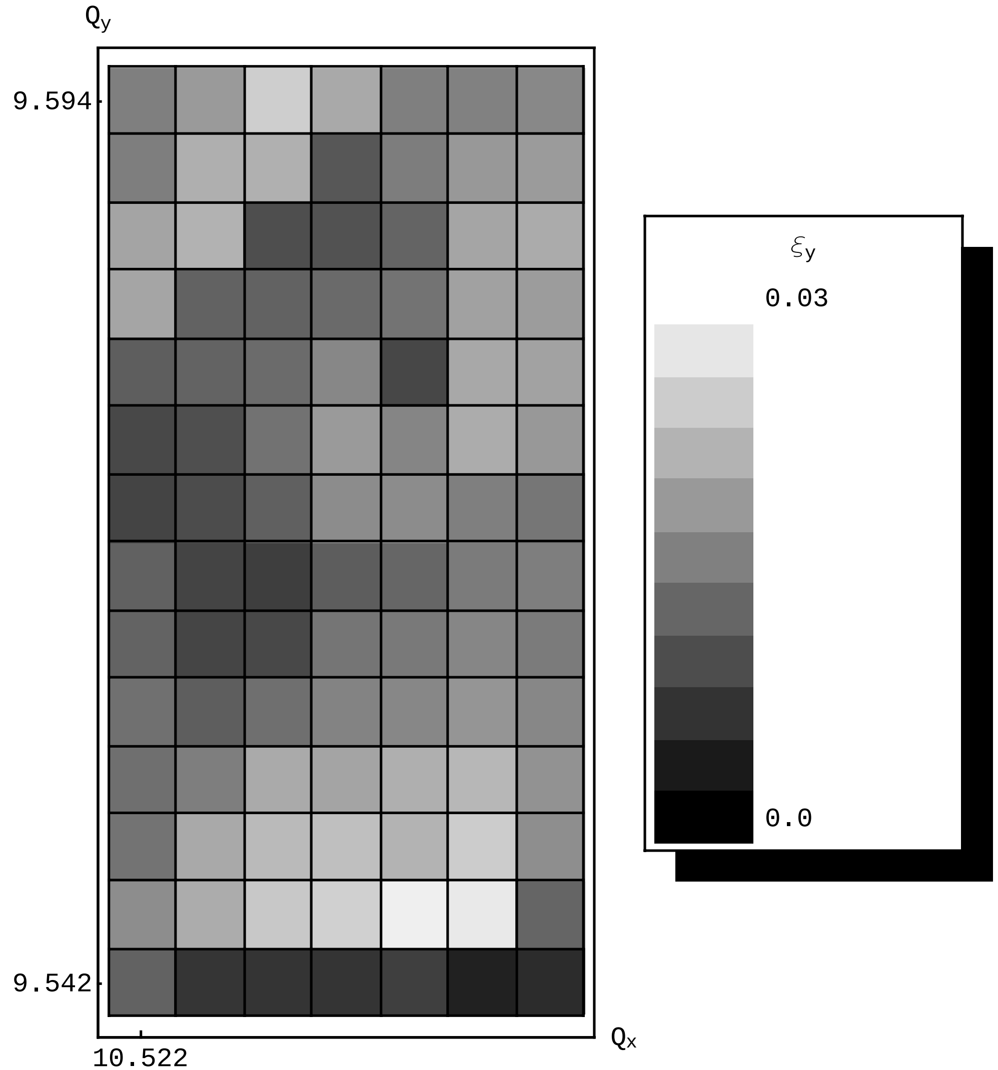


Methods: strong-strong simulations— PIC tricks of the trade

Adaptive Green's function method with longitudinal dynamics (ODYSSEUS)

Simulated luminosity is in excellent agreement with measured luminosity for a well-tuned machine (better than 10%)

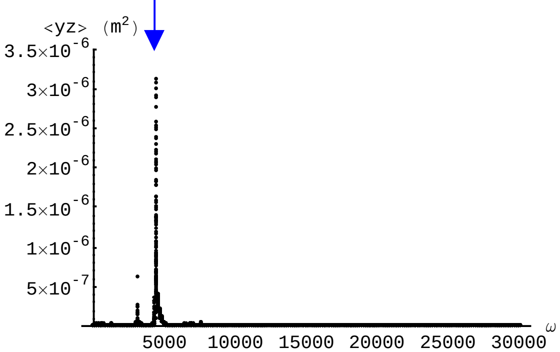
Simulated tune scan, CESR-c, 1.885 GeV, 1 damping wiggler installed, Dec. 21, 2002 conditions.

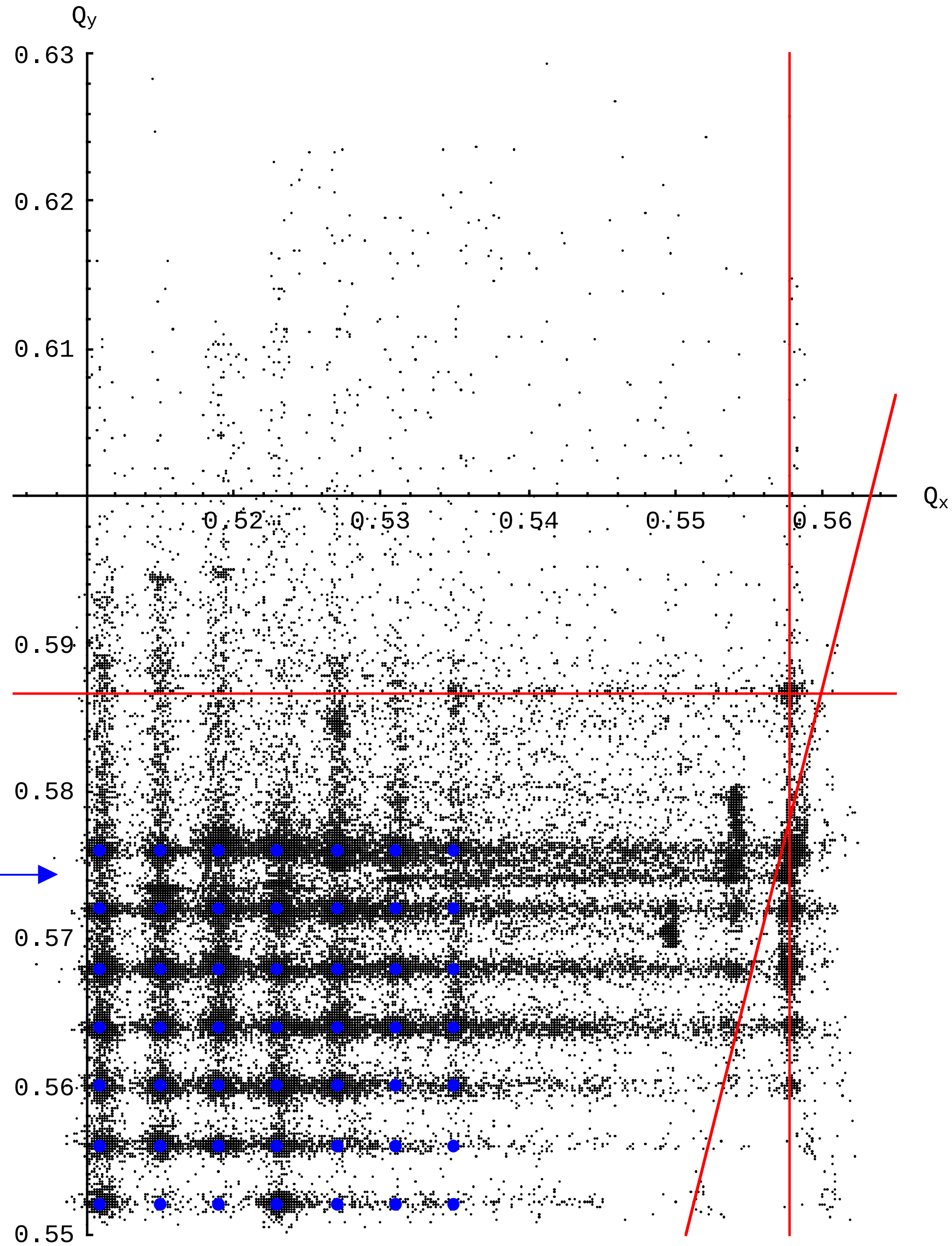


Self-excited coherent motion:

Trajectory of ($\langle x \rangle$, $\langle y \rangle$) coherent signal in transverse tune plane (trajectories superimposed for 7×7 bare machine tunes)

Spectrum of $\langle yz \rangle$: vertical head-tail mode excited





Methods: strong-strong simulations— PIC tricks of the trade

Adaptive Green's function method with longitudinal dynamics (ODYSSEUS, recent work, J. Urban)

- Crossing angle implemented by boost to frame with head-on collisions.
- Includes nonlinear transport through arc lattice.

Methods: strong-strong simulations— PIC tricks of the trade

Poisson solver with reduced grid (Y. Cai, A. Chao, S. Tzenov, T. Tajima, 2001)

• Uses a fast Poisson solver in a region much smaller than the beam pipe to reduce the number of grid points.

• Poisson solver uses FFT and cyclic reduction (FACR).

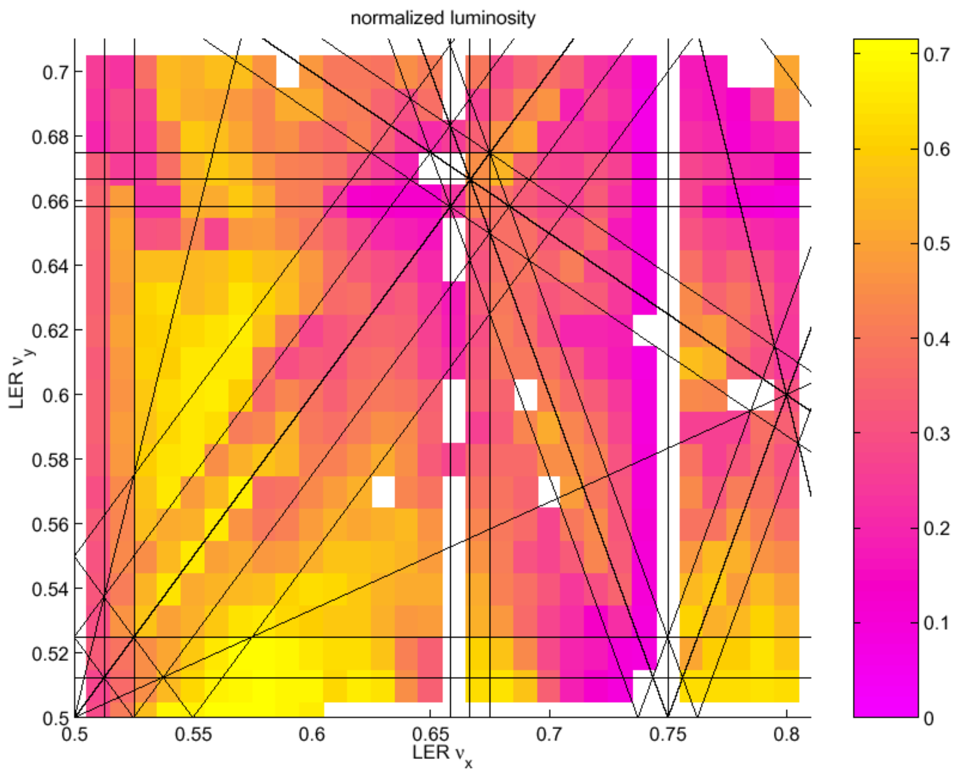
• Before solving the Poisson equation, the potential on the grid boundary is determined by a Green's function method.

Code runs on 32 parallel processors at NERSC.

Recently extended to include longitudinal dynamics, with linear interpolation between adjacent longitudinal slices.

Methods: strong-strong simulations— PIC tricks of the trade

Poisson solver with reduced grid (Y. Cai, *et al.*)

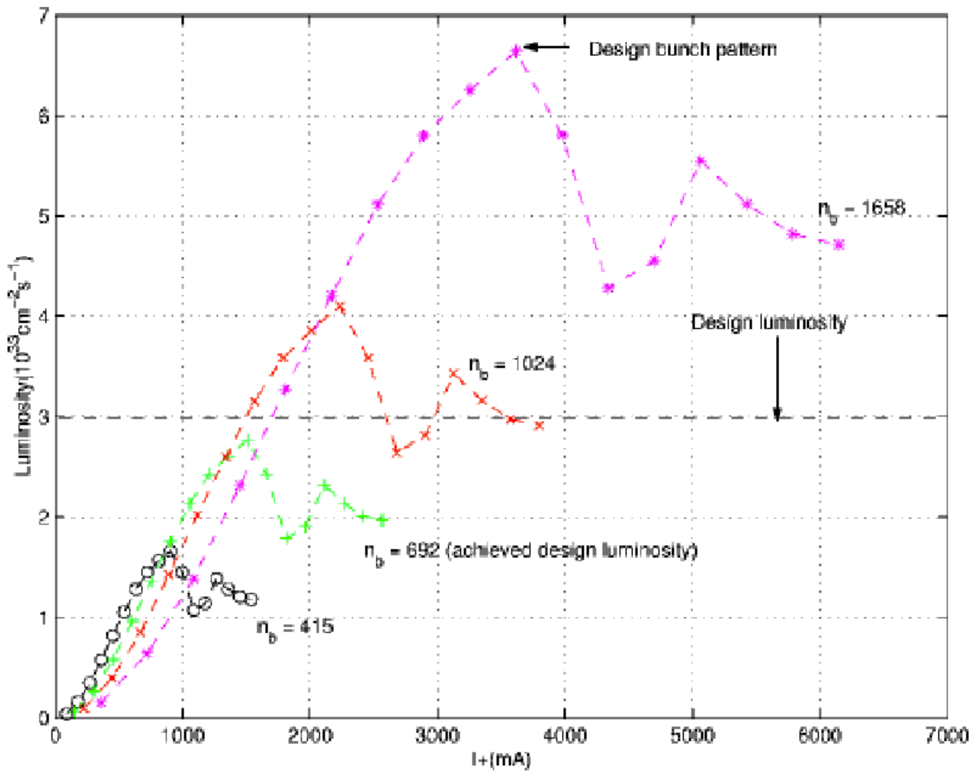


PEP-II LER simulated tune scan. White indicates particle loss (I. Reichel, 2001)

Methods: strong-strong simulations— PIC tricks of the trade

Poisson solver with reduced grid (Y. Cai, *et al.*)

• Simulated beam-beam limit in PEP-II is in excellent agreement (10 - 15% level) with observations.



Methods: strong-strong simulations— PIC tricks of the trade

Green's function method with longitudinal dynamics (K. Ohmi, 2000)

• Convolution of charge density with Green's function is done in Fourier coefficient space.

• Interpolation of potential between longitudinal slices— few slices needed.

• Crossing angle implemented by boost to frame with head-on collisions.

• Simulated luminosity in KEKB is in excellent agreement (15% level) with observations.

Methods: strong-strong simulations— PIC tricks of the trade

Shifted Green's function (J. Qiang, M. Furman, R. Ryne, 2002)

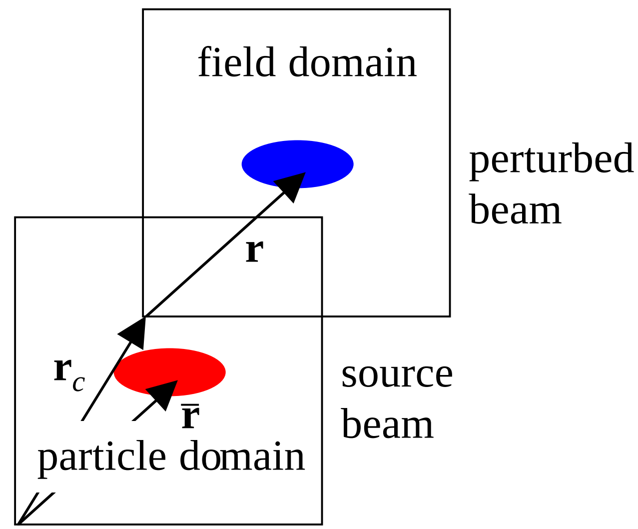
• The standard PIC method, in which the field domain is identical to the particle domain, is inefficient for separated beams (*i.e.*, long-range beam-beam interaction). Most of the grid is empty of particles.

• Make the field domain different from the particle domain by relacing the Green's function

$$G(\mathbf{r},\bar{\mathbf{r}}) = -\ln|\mathbf{r}-\bar{\mathbf{r}}|$$

by a shifted Green's function

$$G_s(\mathbf{r},\bar{\mathbf{r}}) = -\ln|\mathbf{r}_c+\mathbf{r}-\bar{\mathbf{r}}|$$



Methods: Quasi-strong-strong simulations

K. Ohmi, K. Hirata, N. Toge

Method identical to standard weak-strong simulation, but

- after a set number of collisions, the 1st and 2nd moments of the weak beam are calculated,
- the weak beam is replaced by a strong Gaussian beam with the same moments,
- and the strong beam is replaced by a weak beam.

This exchange of weak and strong beams is performed periodically.

Methods: Fast Multipole Method

Has not yet been applied to e^+e^- colliders!

• Force due to nearby source particles is calculated directly.

• Potential due to distant source particles is calculated as a multipole expansion.

Advantage:

Efficient for separated beams.

Disadvantage:

Exaggerated deflections due to close encounters.

Hybrid Fast Multipole Method uses both a grid and a multipole expansion of the fields (Herr, Zorzano, Jones, 2001).

Methods: Numerical Vlasov equation solver

Rarely appled to e^+e^- colliders.

Advantage:

Avoids “noise” due to poor sampling of phase space by macroparticles.

Disadvantages:

Grid exists in phase space. Number of grid points may put too much demand on memory for a 4-D phase space (but see talk by Andrey Sobol this afternoon for a possible solution for uncoupled beams).

Phase space density is not automatically positive.

A Vlasov-Fokker-Planck code for a 2-D phase space with synchrotron radiation excitation and damping (R. Warnock, J. Ellison, 2000) demonstrated the existence of an equilibrium state. It conserves the integral of phase space density to 1 part in 10^5 over several damping times.

Conclusions

• Weak-strong simulations provide useful guidance for accelerator design, choice of tunes, …

• Strong-strong simulations have gained real predictive power, and are fast enough to incorporate longitudinal dynamics. Luminosities predicted to ~10%.

• Extending a FMM code to include synchrotron radiation effects would be straightforward. FMM may be competitive with the shifted Green's function method.

• A number of 6-D strong-strong codes now exist. It's time for a systematic comparison of these codes!

• Comparison with a Vlasov equation solver (a completely different technique) would be desirable.